Indian Institute of Science

BANGALORE

NOTE BOOK

Dr. B. S. Madhava Rao

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2/71. New books Section

(1) Polyominoes by Solomon W. Golomb – The fascinating new recreation in Mathematics.– Shapes made by connecting certain numbers of equal-sized squares, each joined together with at least 1 other square along an edge – Polyomino patterns are actually examples of combinatorial geometry, that branch of mathematics dealing with the ways in which geometrical shapes can be combined – talks of pentominoes, hexominoes etc & solid patterns – full of illustrations galore

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Points for C.C.B. News Letter

(1) An inferential Puzzle : (Math. Gaz. Dec. 68)

Tom, Dick and Harry are married to Jane, Mary and Susan, but not necessarily in that order. Each couple has one child, the childrens' names being Alan, Branda and Charlie.

(i) Mary's child and Harry's child are two stars of the school football team;

(ii) Tom's son is not Alan;

(iii) Dick's wife is not Susan.

Find who are the parents of each child

Solution : ~~from~~(ii)

(a) from(ii) Tom's son is Charlie who from (i) must be Mary's child ie Tom is married to Mary & their child is Charlie;

(b) Harry's child is Alan from(i) and from(iii) & (a) Harry's wife must be Susan and their child is Alan.

(c) From (a) & (b) Dick's wife is Jane.

[Change these to Indian names]

(2) Story about Gauss (Math. discovery by Polya Vol. 1, p. 60 (Blue note book 1/9/69).

(3) Samuel Butler's witty lines of how mathematical ideas come

All the inventions that the world contains
Were not by reason first found out, nor brain,
But pass for theirs who had the luck to light
upon them by mistake or oversight.

(4) Anatole France's comments about teaching mathematics to students:

Do not try to satisfy your vanity by teaching them great many things. Awaken their curiosity. It is enough to open the minds, do not overload them. Put there first a spark; if there is some good inflammable stuff, it will catch fire.

(5) Ramanujan's approximation for π done by him before he went to Cambridge

$\pi = \sqrt[4]{(2143/22)}$, a quantity in error by 10^{-9}.

and $1/2\pi\sqrt{2} = 1103/99^2$ correct to 8 decimal places.

(6) Some strange contributions to number theory by Indians after Ram Ramanujan:

(a) V. Ramesam verified that 2^{71} the quotient of $2^{71}-1$ by the factor 228479 is the product of the primes 48544121 and 212885833 (J.I.M.S. 1912) – Earlier Cunningham had found the factor 228479 of $2^{71}-1$.

(b) Proof of statement of M. Kannan by Sanjana viz

$$20^{45}-1 = 11.19.31.61.251.421.3001.261451.64008001.$$
$$3994611390415801.41994369936162201.$$

(c) ~~R. R. Perasanj~~

(7) Some ~~simple~~ problems of number theory ~~first~~ chosen at random: (Solved & unsolved)

(a) Amicable numbers – Discovered by the Hindus – i.e. Nos m & n are amicable if sum of divisors of $m = n$ and sum of divisors of $n = m$, for eg. 220 & 284: Sum of divisors of 220 (excepting 220 itself) $= 1+2+4+5+10+11+20+22+44+55+110 = 284$ & sum of divisors of $284 = 1+2+4+71+142 = 220$. About 70 such pairs are known, but no proof as to whether the number of such pairs is finite or not.

(b) Density theorem re. primes – Decreases on the average – Question whether there is a last pair of twin primes (for eg. 17 & 19), in view of this decreasing density of primes in general, or whether such pairs exist after any given number

$$5\,mm = \frac{1}{2}\,cm = \frac{1}{2 \times 2.54}\,inch = \frac{1}{5.08} \simeq \frac{1}{5}''$$

* (e) Numerical evaluation of the 18th perfect number by using the IBM. 709

(Ans) This computer the largest perfect number known (1968) and is of the form $2^{3216}(2^{3217}-1)$ & $2^{3217}-1$ is 18th Mersenne prime $-(2^{11213}-1)$ is a still bigger Mersenne prime (1969) – Is $2^{11212}(2^{11213}-1)$ a perfect number ? Yes, Gauss, 6757 digits

however large, has not yet been solved. Fairly large twins exist for eg: 1000427 and 1000429. Converse of density problem viz. whether from any n, we can find n consecutive integers none of which is prime is easily answered in the affirmative for in the n successive numbers $(n+1)!+2, (n+1)!+3, \ldots (n+1)!+(n+1)$ there is no prime, as as $n \to \infty$ such gaps in primes becomes as large as we like.

(8) Fermat numbers (F_n) – $2^{2^n}+1$ – Fermat examined these for $n = 1, 2, 3, 4$ & found them all prime & expressed the belief that it is true for all n. Later Euler showed that for $n = 5$, F_5 is not prime but divisible by 641. Calculations have been made, using computing machines, for $n = 6, 7, 8, 9, \ldots 12$ all giving composite numbers. It is still not known whether there are only a finite ~~for~~ no. of prime F_n's or infinite number & one has no idea of how to approach this problem mathematically. Morehead (1905) showed that $F_{73} = 2^{2^{73}}+1$ is not prime. It is difficult to describe the stupendous magnitude of this number. If each numeral be 5 mm. wide, the number would occupy a length exceeding 5000 trillion (million millions) Km, and in order to write it we should have to cover the equator on the Earth more than a hundred billion times!

9.(a) Abundant numbers ie nos > sum of their divisors eg: $10 > 1+2+5$.

Deficient numbers $<$ " eg: $12 < 1+2+3+4+6$

Perfect numbers $=$ " eg: $6 = 1+2+3$.

Another such number is $28 = 1+2+4+7+14$. Bigger numbers known very early for eg: 496 and 8128. The fifth one is 33,550,336. ~~Every number of the form $2^n(2^n-1)$ is perfect if 2^n-1 is prime.~~ Two such large ones correspond to $n = 2203$ and 2281 and have 1335 and 1373 digits. They all end with 6 or 8 & it is not difficult to prove that every even perfect number ends with 6 or 8 – Euler's theorem that every even perfect number is of the form $2^{n-1}(2^n-1)$ [2^n-1 a prime] – As odd perfect numbers we know nothing, and we don't know if they exist and there is no proof that they don't exist

(e) Mersenne numbers ie primes of the form 2^n-1. This is prime only if n is prime, but if n is prime 2^n-1 need not necessarily be prime (for eg. 3217) – Mersenne numbers leading to perfect numbers – use of electronic computers to find Mersenne nos. – Primes like $2^{1279}-1$ would require more than a century of desk calculator work –

2/71 Collection of notes for Current Science Article on Raman

Report of Ind. Ass. Cultivating Science - 1914-21.

Theory of Vibrations

(1) Report for year 1914 - Bulletin No. 11 was a profusely illustrated monograph by Mr C.V. Raman M.A and contained three papers on the Dynamics of Vibrations. The first was on "the maintenance of combinatorial vibrations by two simple harmonic forces". It described an extremely simple and beautiful method of maintaining combinatorial vibrations of a stretched string devised by Mr. C.V. Raman.

The second described some new phenomena regarding "Motion in a periodic field of force" A new class of resonance was there described in which the frequency of the maintained vibration was a sub-multiple of the frequency of the maintaining force.

The third paper dealt with the vibrations of a bowed strings, and described two new methods of photographically recording the entire motion of a bowed string or that of individual parts on it, and also a simple method of projecting vibration-microscope figures.

All the three papers were very appreciatively noticed and reviewed in "Nature" and "Science Abstracts"

(2) Besides work published in Bulletin No. 11, Mr Raman took up some new lines of research during the year viz. the phenomena of elastic impact and restitution in air & fluids, and an exptl. investigation of the mode of action of the pianoforte hammer

Mention of Raman being elected Vice-President of the Ind. Sc. Congress Association held in Madras, a recognition of the work carried out by him

The session 1914-15 was opened on 7th July with an introductory lecture on "Researches on Elastic impact" by Mr. C.V. Raman. The Hon'ble Justice Sir Ashutosh Mukherjee presided and there was a large and appreciative audience. The lecture was profusely illustrated by slides & noticed by most local newspapers

* Sir Rash Behary Ghosh Barr. of Matter

p. 25 – C. V. Raman's speech seconding resolution re. adoption of the Report of the Committee of Management. "It is several years now, though it does not indeed seem very long, since D first induced the Association – —"

p. 32 – Raja Peary Mohan Mukherjee, one of the leading Zamindars of Bengal was the first M.A. in Science of the Calcutta Univ.

p. 31 – A Braun Cathode Ray Tube and a Capacity Box were handed to Mr C. V. Raman M.A [Woodburn Research Prize] for his valuable research work in the laboratory of the Association.

pp. 31–37 – Sir Gooroo Dass Banerjee's Presidential Address . .

(Sir Gooroo Dass then related how both struggles with obstacles) you observe that if there is any obstacle the loop fastens fast. But when there is an obstacle the loop vanishes. If you invert any obstacle, it causes embarrassment. This was illustrated

Mr. Raman in proposing a vote of thanks said" . . . His explanation of the theory of loops was highly entertaining – – –"

p. 39 – Ganesh Prasad's Presidential Address on the Physico-mathematical section on "Mathematical Research in India" – Ref. to Sir Ashutosh as the first Indian in modern times to do really original work, whose original papers published nearly a quarter of a century ago fired the imagination and shaped the ambition of many a budding mathematician and to whom the Calcutta Math. Soc & Calcutta Univ. owe more than what I can find expression to

Prof. Raman is well known to all of you as an experimentalist of world-wide reputation. But some of you will feel surprised to learn that by his recent mathematical researches he has firmly established his claim to be considered a recent mathematician. The two papers which he read before the Calc. Math. Soc. during the current year are very valuable and I trust he will continue his mathematical researches.

Also refers to Ramanujan at Cambridge.

p. 41 – Short resume by C. V. Raman Esq M.A on Mechanics of Violin bridges – "Mr Raman criticised the views that have been put forward by Helmholtz and De Haas on the subject of the motion of the

violin bridge and said that his observations referred to Helmholtz's theory

p. 45 - Presidential by Mahamahopadhyaya Gurudas Bandyopadhyay (?!)

p. 114 - Asst. Asst for Cultivation of Science (210, Bow Bagan Street) was established in 1876.

p. 117 - Woodburn Prize of 1913 to C.V. Raman M.A

Report for the year 1915 - At the end of Contents, there is a fine portrait of The Hon'ble Justice Sir Ashutosh Mookerjee Kt, Saraswati, Shastra-Vachaspati C.S.I., M.A., D.L, D.Sc, F.R.S.E., F.R.A.Selc. (Vice President)

p. 5 - A short paper by Raman on Intermittent ~~appa~~ Vision appeared in the Phil Mag of Nov. 1915 - Mr Raman also completed an investigation on "discontinuous wave motion" in which he has shown how the mode of vibration of a bowed string discovered by Helmholtz can be experimentally reproduced by another method - A voluminous memoir on the subject - one of the most remarkable developments which may be mentioned here is the discovery of the fact that under certain conditions, the mode of vibration of the bowed string instead of being simply harmonic, may be of a cyclical character - It will appear in Bull. 15 - Ref. to Raman's work by Kalahire & also ref. to Bulletin No. 6 (Experimental investigations on the maintenance of vibrations) in the appearance in the 1914 - reprint of Text-Book on Sound by Prof. E.H. Barton D.Sc., F.R.S. (Macmillan & Co) -

p. 32 - An appreciative reference to their acceptance, Raman's work in the West in Gooroodas Banerjees address.

p. 50 - ref. to the above by Ganesh Prasad

p. 52 - The fact of Ganesh Prasad says Rayleigh is no longer the sole investigator - England, and that a rival has arisen in the horizon (i.e. one Haripada Datta of Univ. of Edin)

pp. 55-63 - Paper by Raman & Ashutosh Dey on "Discontinuous wave motion"

Appendix & Conclusion :- By reading up two discontinuities discussed in the

changes of velocity either simultaneously or in succession, upon a vibrating string which has initially a uniform angular velocity, interesting forms may be obtained which are identical with [a] the motion of a string bowed with suitable pressure close to its centre or at a distance from it not exceeding a quarter of its length, (b) the motion of a string bowed near an end with pressure insufficient to elicit the fundamental pre-eminence, and (c) the successive stages of the forms of vibration obtained by bowing a string nearly near end at the "wolf-note" pitch, or under certain other circumstances in which cyclical [not periodic] vibrations are obtained.

Photographic records confirm the expected analysis of discontinuous vibration forms.

Report for year 1916. (Published 1917)

p. 3 – Profs. Tyndall and White of Univ. of Bristol have recently written to "Nature" acknowledging the correctness of the views put forward by Mr Raman in very flattering terms.

p. 29 – President is Raja Sir Manmatha Mukherjee

Report for 1917 & Proc. of 2nd Science Congress (Published 1918)

p. 5 – one of our Vice-Presidents Prof. C. V. Raman, during the year under review, entered upon the duties of the Sir Tarakanath Palit Prof. of Physics in the Univ. of Calcutta.

pp. 24–28 – Raman announcing the endowment of Dr. Mahendra Lal Sircar Research medal by his son Amrita Lal Sircar.

"I was not fortunate enough to have met and known Dr. Mahendra Lal Sircar, during his lifetime as my first visit to Calcutta was in 1907. But before I end the ten years I have worked in the Institution to which he consecrated his life have enabled me to come into almost physical contact with the personality of the man, and . . . . In my view, a time has come when work of a really important character can be carried out and published exclusively in the Proc. of the Association, and receive adequate recognition from specialists abroad. There are no signs to be seen International barriers in the reading of scientific knowledge. It is our best

* Public Purpose used only for research and not for teaching M.A & M.Sc. Classes.

work that ought to be offered to the Journals and Publications and thus enable them to win adequate recognition".

Chairman (?) awarding the Mathematical Society Gold Medal to S. K. Banerji said: "The medal is a substantial one in more senses than one. 245 Rupees comfortably lies in the body of the medal of gold weighing 15 or 20 tolas. It is hoped unfortunately concerts in it being merged by means of Bonus"

Science Convention 1917.

Presidential Address by Sir Asutosh

p. 1 - Physics - Math. Tech. - President C. V. R., Calcutta Professor. - Addresses on the Progress of Physical Science in Bengal - mentions absence of support from Govt. to start the College of Science although 25 lakhs were collected by private donations - A most grievous disappointment that Govt. support was not forthcoming. Further causes of delay were the law suit on the Palit estate which made the permanency of the endowment a matter for the decision of the law courts, and the attitude of the Member of Education (Sir Harcourt Butler) who decided to permit my joining the Chair until I completely resigned my honorary appointment under the Govt. in the face of this uncertainty. In fact, it was not until July, 1917 that it was possible for me formally to commence my duties as the Head*. The Rashbehary Ghose Professor is still detained in Germany where he was detained for the duration. Absence of State Aid and subsequent difficulty in obtaining apparatus in consequence of the War, the formation of the Physical Laboratory was seriously handicapped - In spite of all this useful research and development of Post-Graduate teaching have been accomplished - Improvement of M. Sc. teaching - Research work, list of own & that of others like S. K. Banerjee, S. K. Mitra, M. N. Shaha & S. N. Bose - Calcutta Physical Society - Our most urgent needs are: further equipment, residential accommodation, and enlargement of careers open to our workers - Govt. should show a more sympathetic attitude in absorbing such men - - - To most men, the knowledge of the degree of recognition that awaits successful work is a considerations not to be despised -

Papers referred to in Appendix :-

(1) on elasticities of Violin tone forced by a Note (I.A.S.V, Patna, Oct., 1917) and a host of other papers by others

After Appendix D - ref. to Raman's Expts with mechanical played violins (Bull. No. 15)

Report for 1918 - Ref. to a joint paper by M.N. Saha & S.N. Bose on a Th Modification of Van der Waals eqn of state (Phil Mag, July 1918)

p. 1 - Parallax in Hindu Astronomy by P.L. Sen Gupta.

Report for the year 1918 [Printed 1920] & 1919.

p. 3 - Ref. to Raman's discovery of the phenomenon of the Coloure of the Striae in Mica.

After p. 50 - Science Convention - Presidential Address by Sir Asutosh

p. 57 - Physico-Math. Section - Presidency Section - C.V. Raman by M.A., Palit Professor. - Opening address. - A short one - At the end: Numerous unsolved problems suggested by the Planetoids still awaited the experiments & mathematician. An outline of a new mathematical theory that was being developed by Raman & his pupils led to very surprising results. A new machine demonstrating the superposition of inharmonic oscillations

p. 73 - Maintenance of Coupled oscillations by Asutosh Roy

p. 75 - on the optical analogue of the Stroboscopic Talbot effect by Bishen Bhushan Roy

p. 79 - maintenance of oscillations by R.N. Ghosh - Refers to Raman's Motor Vibrator for studying the maintenance of oscillations under a variable spring whose period is $\pi/2$ that of the system, where n is any integer (Phys. Rev. Dec. 1912). - Various Cases of resonance for different values of n studied - forced oscillations of attached string under double frequency

Report of Science Convention for 1919.

p. 2 - Study of impact of solids and the phenomena resulting therefrom

by C.V.R. one of the most interesting results obtained being the discovery of a beautiful geometric form of the percussion figures produced on a plane surface by the impact of spheres (Nature, 9/10/1919).

171. pp 2-7 Prof. Raman has shown (in paper to appear in Phys. Review) how Hertz's method can be extended to the problem of transverse impact on a bar or plate

Remarkable acoustic properties of Indian musical drums. It is well known that percussion instruments, generally speaking, give rise to inharmonic overtones & are thus musically imperfect. The Indian Mridanga forms a remarkable exception to this rule — The character of vibrations of the heterogeneous membrane which give rise to these remarkable properties may be investigated by the method of ~~sand~~ sand figures — After greater details on pp. 5 & 6. — Study of the electrically-maintained vibration of wires — Theory of vibrations of bells and gongs —

p. 8 — theory of forced oscillations of strings under damping proportional to the square of the velocity — Vibration of wires maintained by the heating effect of an alternating electric current & study of the applicability of Raman's theory of vibrations — partial tones of bowed stringed instruments — Vibrations of elastic shells partly filled with liquid.

p. 9 — A series of papers relating to research work in optics

pp. 15-17 — Chairman's address — Ref. to Raman accepting the Secretaryship. He is as good, active, devoted and skillful at his experimental table as he is at the Office Desk. His capacity for work is as extraordinary as his versatility is wonderful

p. 37 — Subject of impact of ~~solids~~ elastic solids. In certain cases. It was arranged to study the impact of polished hard steel spheres on steel plates of glass — Produced on a within the glass a percussion figure of a familiar & interesting geometrical form

Report for the year 1920-21 of Science Convocation.

(iii) At annual meeting on (25/5/22) Giving thanks present of is conferred Prof. C.V. Raman M.A., D.Sc & as visitor Ramanathan.

p.4 – Meeting of C.V. Ramanathan being deputed from Trivandrum to do research at Calcutta.

Bulletin of the Association 1–15.

No.6 (1912) – Exptl. investigations on the maintenance of vibrations by C.V. Raman, pp. 1–40.

Subject of lectures delivered 9/1/1909 with Sir Gooroodas Banerjee presiding and on 9/5/12 with Sir Ashutosh presiding

Sections 1 – On a new form of Melde's experiment

" 2 – The small motion at the nodes of a vibrating string

" 3 – The amplitude and phase of oscillations maintained by forces of double frequency.

" 4 – Vibration curves of oscillations maintained by a uniform string

" 5 – The maintenance of compound vibrations by a simple harmonic force

" 6 – Translational modes of motion under variable strings.

References to Lord Rayleigh's Theory of sound – Use of mathematics always accompanied by exptl. results illustrating the math. – different types of maintenance explained by beautiful figures – Differential eqns: $\ddot{u} + K\dot{u} + n^2 u = a\left[P(\cos pt + E_3 - \cos 3pt + E_3') + Q(\cos pt - E_1 - \cos 3pt + E_1)\right.$

Eqns like $u = A_1 \sin pt + A_3 \sin 3pt + A_5 \sin 5pt + A_7 \sin 7pt + B_3 \cos 3pt + B_5 \cos 5pt + B_7 \cos 7pt$.

(1914) No.11 – (1) maintenance of combinational vibrations by two simple harmonic forces.
(by C.V.R.)

(2) On motion in a periodic field of force (mechanical by geometrical paths)

(3) Dynamical theory of the motion of bowed strings

Preface : In Sections VI, VII, VIII & IX the "irrational" moving
subtraction and fraction processes and the obtainable general results
is established that if n, the number of divisor numbers, be a prime integer
greater than unity a true step zig-zag motion is always possible at the
lowest limit available when there is at or near an end of the story, whereas
if n is not a prime, the motion of the traveling point is necessarily of a
more complicated type if it lies outside certain sections of the story

Bull. No. 15 – pp. 1–157 (to be contained) – on the mechanical theory of the vibrations of bowed strings &c and of musical instruments of the violin family, with experimental verification of the results Part I by C. V. Raman (1918) [Huge memoir]

Preface – Hypothetical theory first using analytical mechanics. The qualitative explanation of the effect of a mute on the quality of violin tone is one of the most interesting results of the present investigation – Emphasis laid upon the cases which are of practical interest in music

p. 4 – Eqn of motion $\mu \frac{d^2y}{dt^2} = T_0 \frac{d^2y}{dx^2}$

p. 8 – Ref. to Lord Rayleigh's Theory of Sound – his bible – 1st, 2nd, 3rd & 4th and higher types of vibration dealt with in different Sections

p. 12 – Mathematical theory – Motion of string represented in any actual case represented by the Fourier series

$$y = \sum_{n=1}^{\infty} \sin \frac{n\pi x}{l} \left\{ A_n \sin \frac{2\pi nt}{T} + b_n \cos \frac{2\pi nt}{T} \right\}$$

– Analysis of the motion – question of whether the expression for the force would be a convergent series if n be what value of Σ – If K_n were proportional to a power higher than unity, the expression for the frictional force would not be a convergent Fourier Series and the result would no longer be possible to consider the motion of the string as conforming to infinite series of harmonics – use of relation $\cos \frac{\pi}{7} - \cos \frac{3\pi}{7} + \cos \frac{5\pi}{7} = \frac{1}{2}$ – Eqns

$$\int_{x_0}^{x_0'} \sin \frac{\pi n x}{l} dx \int_0^T F(P_{x,v_{xt}} - v_B) \sin \frac{2\pi x t}{T} dt$$

$$= \frac{K_n T B_n}{2} \cos (V_n + \epsilon_n')$$

& a number of

p. 14? – use of Lagrange's eqns of motion (Ref. to Routh's Advanced Rigid Dynamics & Rayleigh's Theory of Sound) – normal coordinates

Vols 7, 8, 9 conducted by Prof. C. V. Raman M.A. D.Sc.
Hon. Secretary

- Lagrange's eqns for forced vibrations - Use of Young's modulus

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Proceedings 1910 - 1913

Report for 1913 - p. 27 - C. V. R. an assistant Officer of the Geol. Survey of India & an amateur worker in Science & is to be the 1st Prof. of Physics in the Calcutta Univ.

Report for 1912 - Raman reading a paper "on some remarkable properties of the synchronous motor" at Madras Sir Gurudas Banerjee made some complimentary remarks about Raman. (p. 32) Quite interesting for eg. experiments putting Nature to torture sometimes

Proceedings Vol 1-3 -1917 - Papers of Raman

" Vol. 6. 1919 - None by Raman,

Vol. 7 - 1921-22 . p. 159 - On whispering galleries by C. V. Raman

[p. 160 - Ref. to Raman being in England in 1921

(acoustical phenomenon explained on basis of velocity of sound and structure of the domes)

Vol. 8 - 1923 . - No. of papers by K. R. Ramanathan (Asstt. Lecturer in Physics, Univ. College, Rangoon & in 1923) & one in collaboration with Raman.

Vol. 9 - 1925 - 26 . p. 271 - on the total reflection of light by

C. V. Raman F.R.S (when?)

- Uses Bessel function formulae due to Sommerfeld and also ref. to Nicholson - Lamb's formula using Bessel functions

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Intro. to Modern Physics by Richtmyer, Kennard & Lauritsen

p. 330 - The Raman effect - Kramers & Heisenberg (Z. Physik, 31, p. 681, 1925) had shown that according to classical electromagnetic theory, if the scattering electrons in an atom or molecule are in motion, the scattered light should contain other frequencies in addition to that of the incident light - Failure of

notice that explanation of the classical theory has been due to the common assumption that the radiating particles were at rest except as disturbed by radiation. In 1928, independently of the theoretical prediction, Raman and Krishnan discovered the phenomenon experimentally [Nature, Vol. 121, p. 501, 1928; & Raman, Ind. J. Phys. Vol 2, p. 387, 1928]. The scattering of light with a change in its frequency is commonly called the Raman effect. In this, it is frequency shifts in the scattered spectrum that are determined by the nature of the scatterer rather than by the frequencies themselves.

The explanation in terms of quantum theory is very simple. When a photon of frequency ν is "scattered" by an atom or molecule whose quantum state is not altered in the process, the scattered photon has the same frequency as the incident photon. But it may happen that the atom or molecule is changed in the process from a state of energy W_1 to a state of energy W_2. Conservation of energy requires that the frequency ν' of the scattered photon be correspondingly altered

$$h\nu' + W_2 = h\nu + W_1 \quad \text{ie}$$

$$\nu' = \nu + \frac{W_1 - W_2}{h} \quad \text{or simply} \quad \nu' = \nu + \nu_{12}$$

(ν_{12} = frequency of some characteristic emission or absorption line). In terms of classical ideas the Raman effect has been predicted, tentatively, by Smekal.

Proceedings of Ind. Assn for Cultivation of Science now Vol 10. has been transformed to Ind. J. Phys. Vol I – Conducted by C.V. Raman F.R.S.

(Vol. 10) Vol 1. – Ind. J. Physics (1926-27)

(Vol. 11) Vol 2 – (1927-28)

p. 387 – A new radiation by Prof C.V. Raman F.R.S

(Inaugural Address delivered to the South India Science Association on Friday the 16th March, 1928, at Bangalore)

ie announcing the Raman effect

Conclusion – The line spectrum of this radiation was first seen on 28/2/1928 [ie 43 years ago tomorrow]. The observations were given publicity the following day.

(Vol 12) Vol. 3 · p. 357 – A classical derivation of the Compton Effect by Sir C.V.

Again based on vibrations a (Modern Analyst) film titled how the most arbitrary type of wave-disturbance can be represented as the superposition of plane trains of waves travelling in all directions in space.

Vol. 4, p. 281 – Bibliography of 150 papers on the Raman Effect by A. S. Ganesan – Again a vibration of the Raman effect

Vol. 6, 1931, p. 263 – Sir C.V.'s Nobel Lecture – Calls Raman effect as the optical analogue of the Compton effect.

p. 353 – Experimental proof of the wave theory of light by Raman & Bhagavantam – This is what Sir C.V. came to ask me sometime back – The whole paper looks ridiculous for a Nobel laureate and his idiotic collaborator. II

Vol. 7, p. 431 – Again Bibliography of Raman Effect 1930-34 by ~~N.C. Sinha~~ S.C. Sirkar

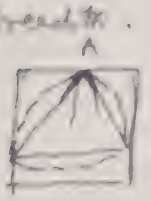
Vol. 8 – Contributed by Sir C.V. Raman Kt, M.A., D.Sc, LL.D, F.R.S, N.L (!) and Prof K. S. Krishnan D.Sc. (1934-35)

(Vol. 10) Vol. 9 – Edited by D. M. Bose, B. N. Ghosh & S. K. Mitra

p. 553 – Bibliography of Raman Effect III by Sirkar & D. Chakravarty

3/71 Matter for CCB · Newsletter (Contd)

(8) Rotors in polygons & polyhedra – for eg. a rotor in a square is a curve which may rotate within it remaining tangent to all sides of the square. This is a generalisation of the earlier well-known curves of constant breadth. For the square the figure formed by the arcs having centres at A, B, C (the vertices of the equilateral Δ^{le} of A = length of a side) & make $\angle BAC = 60°$ & pass thro B & C respectively is a rotor.



(9) Calculation of π to 100,000 decimal places

(71) News Books Section

(1) The Ambidextrous Universe by Martin Gardner. (The Sc. American Math. Puzzle Tackling man) - Allen Lane The Penguin Press, London 1967
- Very interesting ~~book~~ popular book on elementary particle physics

(2) Riemann Surfaces by Ahlfors and Sario (Princeton Univ. Press, 1960)
- Not like the old book by Eisenhart.

Chap 1 - Surface topology - Topological spaces, Surfaces, Covering surfaces, simplicial homology, Singular homology, compactification, The classification of polyhedrons, Existence of triangulation (8 articles)

Chap 2 - Riemann Surfaces - Defns & constructions, Elem. Theory of fns on Riemann surfaces, Dirichlet problem & applns (3 articles)

Chap 3 - Harmonic fns on Riemann surfaces - 5 articles

Chap 4 - Classification Theory - 8 articles

Chap 5 - Differentials on Riemann surfaces - 4 articles (ending with the Riemann-Roch Theorem)

(3) Group Theory by E. P. Wigner - Translated from German - Expanded & Improved Edition - Academic Press, 1959.

(4) The theory of groups by Jan. D. Macdonald - Oxford, Clarendon Press, 1968
- Elementary book with Examples

(5) Space through the ages by C. Lanczos - Dublin Inst. Academic Press, 1970
- Evolution of geometrical ideas from Pythagoras to Hilbert and Einstein.

(6) Mathematics: Creation and Study of forms by Jacqueline P. Evans - Addison-Wesley, 1970
- A good elementary book

(7) Topics in harmonic analysis - Related to the Littlewood-Paley Theory by Elias M. Stein - Annals of Math. Study - Princeton Univ. Press, 1970
- In references to the Groups the reference is Harish-Chandra

(8) Modern mathematics for elementary teachers by Wagner & Wolf
International Text-Book Co - Scranton, Pennsylvania - 1965

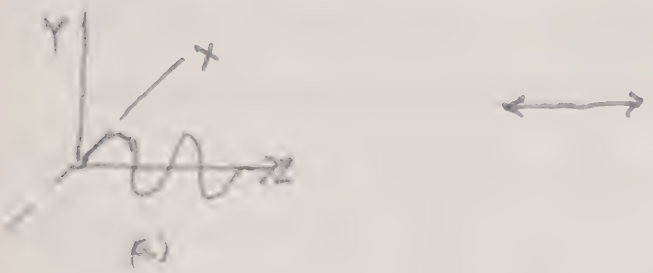
(* This is #7)

(9) Polarised light by Shurcliff & Ballard* – Van Nostrand Co. Princeton N.J. p. 83 – It is becoming increasingly clear that light and matter belong ultimately to the same family. The family has no name, but it is sometimes referred to as the set of fundamental particles. It includes more than 30 members – listed for eg. by Frisch & Morrison in Momentum Book (Van Nostrand Momentum Book #1) on Elementary Particles. It includes well known particles electrons, protons, and neutrons, and also various mesons, pions, kaons, and so-ons (!). Every one of these particles has energy, momentum, spin, upper limit c on speed, the capability of exhibiting wave-like properties, the capability of interacting with other members of the family and the capability of being suddenly created or suddenly destroyed. The photon has every one of these properties and must therefore be recognised as a member in good standing of the same family.

The photon's claim to membership is clinched by the statement that its spin [or A·M (should be intrinsic)] is simply related to that of the electron. The electron has A·M of magnitude $h/4\pi$, and a circularly polarised photon has exactly twice the A·M – an arrangement that is highly appropriate in that when an electron reverses its spin direction while emitting a photon, the photon can carry away exactly the necessary amount of A·M so that this quantity is conserved. "Spin-wise", the photon is as closely related to the electron as a key to a lock. Actually, the exact 2-to-1 match of photon spin applies not only to the electron, but also to most other matter-particles. It is noteworthy, and indeed almost incredible, that the match is valid whether the energy of the photon is great or small. Photon energies may run a majestic range, but the spin of the photon is as unchanging as the Rock of Gibraltar.

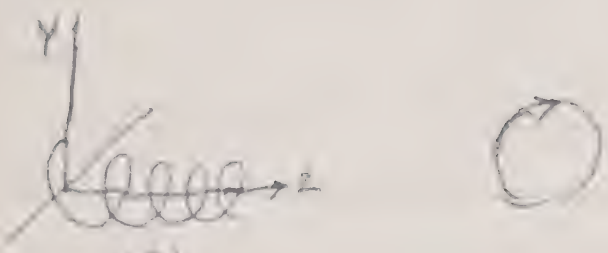
(In view of Raman & Bhagavantham's original paper in Proc. I. Phys on spin of the photon, the above explanation which I gave to Raman, when he came home to tell us about it, is very simple.)

- Similarity & function of the photon and the gluon; call 'gluon-tearing'
- Many interesting remarks on pp. 83–85
- Def^n of polarized light on p. 9 (Scattered patterns) – very quick and simple diagrams (polarized light = light where the oscillation takes a simple pattern)

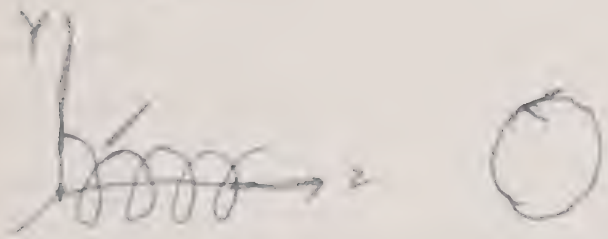


(a)

(b)



(c)



Observer situated to the right on the +ve Z axis, looking towards the light source at the origin

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3/71 Ivar Holks Sechin

(1) A book on Cobordism theory – Princeton 17-12. Series – Related to Homotopy theory

(2) Proc. Int. School of Physics "E. Fermi" Course 45, 1969 on Local Quantum theory (1) edited by R. Jost (Academic Press)

– mostly by mathematicians – gives details in the description of a "revival of mathematical physics that has taken place over the past decade and a half in the form of developments in modern science that I personally appreciate most. Mathematicians as well as physicists have had to travel great distances and overcome annoying obstacles in order to gather once again for fruitful collaboration. Extensive testimony of such a meeting is recorded in the Report of Course VIII on Methods of elementary particles.

Contributions:–

(1) Algebras of operators by J. Dixmier – Initial C* algebra – Systems of Heisenberg

(2) Introduction aux équations aux derivées partielles by H. Malgrange.

(3) Local quantum theory by H. Araki – Basic axioms (including a postulate for quantum mechanics) – Scattering theory – Analyticity – Algebraic aspects.

(4) Models of quantum field theory by J. Glimm. – Fock space

(5) Constructing the $\lambda(\phi^4)_2$ theory (?) by A. M. Jaffe

(6) Euclidean quantum field theory by K. Symanzik with an appendix by S. R. S. Varadhan on p. 219 (Is he an author?)

(7) Integration in function space by J. Glimm

(8) Operator algebras – by M. Guenin

(9) Current Commutation relations etc by R. Streater.

Q 11 on Fourier series from Advanced Calculus by A. E. Taylor

(a) Define the Fourier series corresponding to a function $f(x)$, assuming that the series converges and that $f(x)$ is integrable in $(-\pi, \pi)$; its Fourier coefficients satisfy the inequality

$$\frac{a_0^2}{2} + \sum_{k=1}^{n} (a_k^2 + b_k^2) \leq \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)^2 \, dx.$$

(b) and sketch briefly the essential points involved

$$
\begin{array}{r}
216 \\
108 \\
\hline
1296
\end{array}
$$

$6^{2^3} = 6^8 = 36 \times 36 \times 36 \times 36$

$$
\begin{array}{l}
= 1296 \times 1296 \\
\hline
\quad 7776 \\
\quad 11664 \\
\quad 2592 \\
\quad 1296 \\
\hline
1679616 + 1 = 1679617
\end{array}
$$

$$
\begin{array}{r}
17 \overline{)\,1679617\,(\,988001} \\
153 \\
\hline
149 \\
136 \\
\hline
136 \\
136 \\
\hline
017
\end{array}
$$

* by Hans Riesel.

in proving the convergence theorem of the Fourier series.

(b) If $f(x) = x$, and $f(x) = |x|$ $(0 \le x \le \pi)$, find the sine series and its cosine series respectively of the two fns respectively.

$$\left[2\left(\frac{\sin x}{1} - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \cdots \right) \right.$$

$$\left. \frac{\pi}{2} - \frac{4}{\pi}\left(\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \cdots \right) \right.$$

(b) (v) Find the sine and cosine series respectively of the functions $f(x) = x$, and $f(x) = |x|$, $(0 \le x \le \pi)$.

(C) Derive the result

$$\pi \cot \pi x = \frac{1}{x} + \sum_{n=1}^{\infty} \frac{2x}{x^2 - n^2}$$

by a suitable Fourier expansion.

9|3|71 Mathematics of Computation — Vol. 23, No. 105, Jan. 1969.

(1) p. 413 - Some factors of the numbers $G_n = 6^{2^n} + 1$ & $H_n = 10^{2^n} + 1$ *

- in analogy with the Fermat numbers, the above type of nos have been examined for primality, and determination of factors when it is composite (for values from $n = 1$ to 29 for G_n & $n = 1$ to 29 for H_n) using the Computer TRASK of Sweden.

Results from the tables: G_0, G_1, G_2 are prime, $G_3 = 98801 \times 17$, and all others up to G_{29} are composite. G_6 is composite, but factors could not be found. Such for several other n, only one factor could be found,

H_0, H_1 are primes, H_2 to H_5 are completely factorised & H_{10} is the smallest H_n with unknown character. H_8 proved composite, but factors not found.

(2) p. 423 - A note on Reviews of four tables by M. del. W. Powell, and W. J. Blundon.

A search for the integral solutions of the Diophantine

equation $x^3 + y^3 + 2z^3 = K$ for every $|x|, |y|, |z| < 10^5$ was made on an IBM-1620 Model I. These results showed that there were just 19 values of K in the range $1 \leq K \leq 999$ for which no solution is known.

On this model, a fortran ~~model~~ program for this problem took about 15 hours to run. Later on we rewrote the program in the assembler language which ran about 15 times faster, and decided to extend the search to 10^4. This required a computing time of 1000 hours, and the run was made over a period of one year on a low ~~[illegible]~~ priority basis. This extended search gave 20 more values of K given in Table I [one example from this table for X, Y, Z, K is

$$643 = (2926)^3 + (4405)^3 + 2(-3809)^3$$
$$= (7210)^3 + (7939)^3 + 2(-7592)^3 \quad !\ ]$$

Another example is $958 = (6220)^3 + (9268)^3 + 2(-8033)^3$!]

As a result we have just 19 values of K in the range $1 \leq K \leq 999$ for which no solution of $x^3 + y^3 + 2z^3 = K$ is known at this time given in Table 2, are

76, 148, 183, 230, 253, 356, 418, 428, 445, 482, 491, 519, 580, 671, 734, 788, 923, 931 and 967.

(3) Review of the collection of papers on Computers in Mathematical Research (Ed. by Churchhouse & Herz) – North-Holland, [illegible] 1968 (Imposing list of items of use of computers).

(4) p. 539 Relatively prime odd amicable numbers by Peter Hagis, Jr.

Whether or not a relatively prime pair of amicable numbers exists is still an open question. In this paper some necessary conditions for m and n to be a pair of odd relatively prime amicable numbers are proved. In particular, lower bounds for m, n and mn and the number of prime divisors of mn are established. The arguments are based on an extensive "enhanced" case study carried out on the CDC-6400 at the Temple University Computing Centre.

Two positive integers m and n are said to be amicable if

$$\sigma(m) = m + n = \sigma(n)$$

where $\sigma(k)$ is the sum of the positive divisors of k. To date, almost 4000 pairs of amicable numbers have been found, none of which is relatively prime. Rosolde has shown that if m and n are relatively prime amicable numbers, then $mn > 4 \cdot 10^{46}$. The author has shown earlier that if m and n are relatively prime amicable numbers of opposite parity then $mn > 10^{74}$. The present paper Rosolde's result — Proof of additional theorem — Corollary. If m and n are relatively prime odd amicable numbers, then $m > 10^{33}$ and $n > 10^{33}$.

This improves Rosolde's result that ~~$n > m > 10^{23}$~~ $n > m > 10^{23}$. It can also be remarked that the word "odd" can be removed in the Corollary, for it has shown by the author earlier that if m and n are relatively prime amicable numbers of opposite parity, then $m > 10^{36}$ and $n > 10^{36}$.

(5) p. 545 **On divisibility by nine of the sums of the even amicable pairs by Elvin Lee**

Most known even ~~numbers~~ amicable pairs have sums divisible by 9 [Reference to Martin Gardner's article in Sc. American, V. 218, No. 3, 1968, pp. 123-24 re: "A short treatise on the useless elegance of perfect numbers and amicable pairs"]. The general form of the exceptions to the rule of divisibility by nine (Gardner's rule) is deduced, and the results expressed in the form of a theorem. A computer search based on a corollary to the theorem is described and six new exceptions to Gardner's rule are found.

Lemma 1 - Neither member of an even amicable pair is divisible by three.

Theorem - Quite complicated

A corollary to the theorem — also complicated.

639. (6) **Chebyshev polynomial expansion of Bose-Einstein functions of orders 1 to 10**

The most general Bose-Einstein function is given by

(b) integral representation

$$B_p(\eta, u) = \frac{1}{\Gamma(p+1)} \int_0^u \frac{x^p \, dx}{e^{x-\eta} - 1}$$

where η and u may be complex

8/8. Sinesha Brudno - M.9.7, Mass - new results on equal sums of like powers

$$275^6 + 36^6 + 179^6 = 65^6 + 276^6 + 169^6$$
$$211^6 + 125^6 + 300^6 = 68^6 + 289^6 + 249^6$$
$$1^6 + 515^6 + 500^6 = 556^6 + 197^6 + 409^6$$
$$148^6 + 249^6 + 103^6 = 188^6 + 243^6 + 11^6$$
$$539^6 + 412^6 + 643^6 = 497^6 + 652^6 + 449^6.$$

Ref. to K. Subba Rao - on sums of 6^{th} powers - J. Lond. Math. Soc. 9, 1934, p. 173.

(3|71) New Books Section

(1) ✓✓ Mathematics on vacation by J. S. Madachy. [Thomas Nelson & Sons]

Very interesting book with all kinds of new types of ~~recreations~~ mathematical recreations.

Contents :

(1) Geometric Dissections
(2) Chessboard placement problems
(3) Fun with paper
(4) Magic, antimagic & talisman squares
(5) Puzzles & Problems
(6) Number recreations
(7) Alphametics
(8) Conglomerate

Under (5) — (a) The commoner's dilemma about the white and the black pea is very interesting

(b) Construct the smallest equilateral Δ^{le} inscribable in a square, and construct the largest equilateral Δ^{le} inscribable in a square. Show, by simple means, that the Δ^{le} so constructed are indeed the

$5^8 = 25 \times 25 \times 25 \times 25$

$= 625 \times 625$

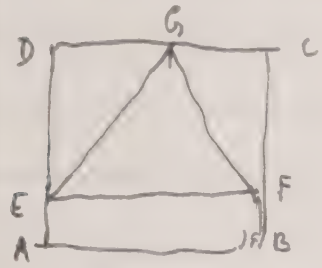
3125

1250

3750

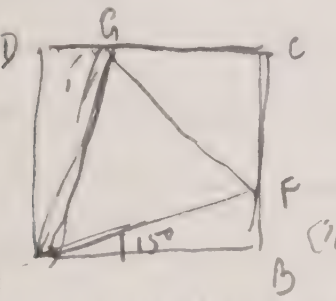
390625

smallest & largest possible. If the square is a unit square, calculate the lengths of the sides of the inscribed Δs



(i)

[1st case, $EG = FG = AB$



(ii)

2nd case, AF, AG make angles of 15° with AB & AD respectively & E coincides with F.

(c) Number toughies:

(i) what two whole numbers, neither containing any zeros, when multiplied together equal exactly 1,000,000,000.

(ii) Similarly for 1,000,000,000,000,000,000

(i) No. $= 10^9 = (2^9)(5^9) = (512)(1,953,125)$

(ii) No. $= 10^{18} = (2^{18})(5^{18}) = (262,144)(3,814,697,265,625)$

(other connected results).

3/71 The easiest way to find such solutions is to find all powers of 2^n containing no 0's and find the corresponding values of 5^n having no 0's. Examination of multiplication tables (in a standard work of Mathematical Tables published by the Chemical Rubber Publishing Co.) shows that the following powers of 2 less than 100 contain no 0's: 1, 2, 3, 4, 5, 6, 7, 8, 9, 13, 14, 15, 16, 18, 19, 24, 25, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 41, 42, 43, 46, 47, 49, 51, 52, 53, 54, 56, 57, 58, 59, 61, 62, 63, 64, 65, 67, 68, 69, 71, 72, 73, 74, 76, 77, 78, 79, 81, 82, 83, 84, 86, 87, 89, 91, 92, 93, 94, 96, 97, 98, 99. Only a few of these are matched by corresponding powers of 5^n containing no 0's:

$10^1 = (2)(5)$

$10^2 = (2^2)(5^2) = (4)(25)$

$10^3 = (2^3)(5^3) = (8)(125)$

$10^4 = (2^4)(5^4) = (16)(625)$

$10^5 = (2^5)(5^5) = (32)(3125)$

$10^6 = (2^6)(5^6) = (64)(15625)$

$10^7 = (2^7)(5^7) = (128)(78125)$

$10^{18} = (2^{18})(5^{18}) = (262144) \times (3814697265625)$

$10^{33} = (2^{33})(5^{33}) = (8589934592) \times (116415321826934\,814453125)$

Rudolph Matthijsen of Holland City, New Jersey, checked [illegible] including 2^n up to 2^{5000} and found no powers of 2 greater than 86 that did not contain zeroes. Fred Greenberger, a mathematician at the Rand Corporation in Santa Monica, California, programmed an IBM 1620 electronic computer to produce consecutive powers of 2 and type the exponent for those that lacked zeros. The machine had [illegible] natural logarithms. It went to 2^{10535} before the computer had to be [illegible] random powers showed [illegible] in [illegible] where an absence of zeros. The number 2^{10535} has 3172 digits and these digits pass the frequency test of randomness, so that there are about 317 each of 0's and 5's in the number. This empirical results indicate that the probability of a power of 2 higher than 86 having no 0's is quite low. The matter awaits an analytical proof, but the above [illegible] tends to discourage further empirical searching.

For those interested in further empirical results, T. Charles Jones, a student at Davidson College in Davidson, North Carolina reports that up to 2^{57134} (which has 7200 digits) the results are equally negative. And, I can add, even if a power of 2 greater than 86 is found which contains no zeros, the probability that the corresponding power of 5 would also contain no 0's is fantastically small.

(d) A problem in confusion:

When Bert was just one year younger than Bill was when Ben was half as old as Bill will be 3 years from now, Ben was twice as old as Bill was when Ben was 1/3 as old as Bert was 3 years ago. But, when Bill was twice as old as Bert, Ben was 1/4 as old as Bill was one year ago.

Ignoring odd months and considering that Bert has passed the half-century mark, find how old these three friends are

Ans:- Bert is 72; Ben is 80 & Bill is 85

—

(c) The oracle of the three Gods

A certain oracle is presided over by three gods, who take turns in answering the questions put to it by pilgrims. The three gods are the God of Truth who always tells the truth; the God of Falsehood who always lies; and the God of Equivocation, who alternates tells the truth and lies.

One day a pilgrim arrives and wishes to know whether or not his wife is faithful. Unfortunately, the pious pilgrim does not know which god will answer his question. Moreover the priests of the oracle allow no more than one question from each pilgrim.

How should the pilgrim state his question, so as to be sure of either his wife's fidelity or unfaithfulness?

Solution :- An amazing variety of answers to this puzzle are possible. Here are rather two simple questions either of which would have served the pilgrim's purpose.

(i) If I asked you if my wife is faithful, would you say yes?

(ii) Are you as honest as my wife is faithful?

(i) Let us be analyzed as follows. If the pilgrim's wife was faithful the God of Truth would answer "yes". The God of falsehood, if merely asked "Is my wife faithful?" would say "No" (if the wife was faithful this god would lie about it). But this god is asked what his answer to such a question would be, and, true to his calling, he falsely says "yes" The God of Equivocation, now no matter whether it is his turn to tell the truth or to tell lies, would likewise answer "yes" to the pilgrims question.

In other words, if the answer to the first question is yes, then the pilgrim's wife is faithful. Conversely, if the pilgrim's wife is unfaithful, all the gods would answer "no" to the question.

(ii) A similar analysis for the second question yields the same result: A yes by the Gods means the wife is faithful, a no means she is not

(5) Did the butler do it?

"Where are those valuable Indian-head pennies I left on the table this morning, James?" "I put them in a square array and now there are only two left. You didn't take them, did you?"

"Well, sir," replied the butler, "shortly after you left, three burglars came in. They divided the pennies equally among themselves, but left the two because they could not divide them equally."

Is James telling the truth?

Solution: The butler lied when he said that three burglars left two pennies after dividing the rest evenly. There is never a remainder of 2 when a square number is divided by 3. All integers may be placed in one of three categories, $3K$, $3K+1$ or $3K+2$ (K any integer). Their squares are $9K^2$, $9K^2+6K+1$ and $9K^2+12K+4$ respectively. The first leaves no remainder, while the other 2 leave a remainder of 1.

(9) A travelling man

A traveller sets out to cross a desert. On the first day, he covers $1/10$ of the journey; on the second day, he goes $2/3$ of the distance traveller the first day. He continues on in this manner, alternating the days on which he travels $1/10$ of the distance still to be covered, with days on which he travels $2/3$ of the distance to the distance already covered. At the end of the 7^{th} day, he finds that $22\frac{1}{2}$ miles more will see the end of his journey.

How wide is the desert?

Solution - Calculation yields $\frac{3x}{16}$ miles remaining on the 7^{th} day, where x = width of desert. Since $\frac{3x}{16} = 22.5$ miles, $x = 120$ miles.

(h) Major Perkins

Assuming the truth of the following statements, what can you deduce about Major Perkins?

only [illegible] soldiers [illegible] may be absent with a [illegible]

No bleary-eyed military men are all musical.
Men who never try to shave with a tooth brush never play golf.
Major Perkins plays the tuba on Wednesdays.
Only musical people who are golfers forget to change their socks.

Solution: Many solutions to this logical problem include the assumption that Major Perkins was a man. If so, the logical deduction from the statements in the problem is that he is musical, not bleary eyed, not a golfer, does not try to shave with a tooth brush and he remembers to change his socks.

Major Perkins could have been a female Major, in the Salvation Army. Actually the only completely valid conclusion that can be reached from the statements as given is that Major Perkins (~~sex~~ sex not determined) plays the tuba on Wednesdays.

Under (6) ie Number recreations. — I hope here to avoid repetition of often-published anecdotes. Familiar material is examined from a somewhat different viewpoint, and a generous selection of new work or results is also included.

(13/7/71)

(a) Perfect and multiperfect nos: If sum of divisors of N is equal to a multiple of N, N is called multiperfect (eg. divisors of 120 are 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40 and 60 and their sum is 240). There are other numbers whose divisor sums are equal to 3 or 4 times the numbers themselves — Amicable nos (for eg. 220 & 284) If, after more than 2 steps in the process of successive additions of the divisors of a number, the original one is obtained, the number is called a Sociable number. The divisors of 12496 are 1, 2, 4, 8, 11, 16, 22, 44, 71, 88, 142, 176, 284, 568, 781, 1136, 1562, 3124 and 6248 and their sum is 14288. There are 19 divisors of 14288 and their sum is 15472. This has 9 divisors and their sum is 14536. This has 15 divisors whose sum is 14264. This again has 7 divisors whose sum is 12496 the original number. It is interesting to find that there is only one

* $22 \to 1+2+11 = 14$

$14 \to 1+2+7 = 10$

$10 \to 1+2+5 = 8$

$8 \to 1+2+4 = 7.$

$7 \to 1.$ ie $s^4(22) = 1.$

$138 = 1 \cdot 2 \cdot 3 \cdot 6 \cdot 23$

$2^3 - 1 \;)\; 2^{15} - 1 \;(\; 2^{12} + 2^9 + 2^6 + 2^3 + 1$

$2^{15} - 2^{12}$

$2^{12} - 1$

$2^{12} - 2^9$

$2^9 - 1$

$2^9 - 2^6$

$2^6 - 1$

$2^6 - 2^3$

$2048 = 2^{11}$

23×89

207

184

2047

other sociable number is known, 14316. This number also ends in 6 & after 28 steps.

We can write in short $s^5(12496) = 12496$, $s^{28}(14316) = 14316$ and for an amicable number N, $s^2(N) = N$. If $s(N) = N$, the number is perfect. The number 138 which yielded 32,185,715,586 after 125 steps may produce a continually increasing series of divisor-additions, in which case it could be represented by $s^\infty(138) = \infty$. If 138 is sociable $s^k(138) = 138$ and k is to be found. The only other possibility is 138 may eventually yield a prime number, thus ending the series with 1. The number 22 produces the prime 7 after 4 steps;* thus $s^4(22) = 1$. For a prime p, $s(p) = 1$.

(a) Perfect and multiperfect numbers: The first five perfect numbers are 6, 28, 496, 8128 and 33550336. Until 1952 only 12 perfect nos. were known & the problem was at a level impracticable for desk calculation. Then SWAC, the National Bureau of Standards' Western Automatic computer found the next five perfect nos. More were found subsequently & by 1965 23 perfect nos. were known. All even perfect numbers are of the form $2^{p-1}(2^p-1)$ where p and (2^p-1) are primes, the latter being the Mersenne prime M_p (if p is not prime, 2^p-1 is not prime, and if p is prime it does not follow that 2^p-1 is prime). The only known values of p (1968) that yielded Mersenne primes are: 2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107, 127, 521, 607, 1279, 2203, 2281, 3217, 4253, 4423, 9689, 9941 and 11213. There are undoubtedly many more beyond $p = 11213$. The size of these primes grows rapidly with p. Thus M_{39} has 12 digits, M_{1279} has 386 digits and M_{11213} (the largest known prime, in fact) has 3376 digits.

The perfect nos. $2^{p-1}(2^p-1)$ are bigger than M_p's. The perfect no. corresponding to $p = 23$ has in $2^{11212}(2^{11213}-1)$ has 6751 digits. Table gives of 1st 23 perfect nos (denoted by V) – The actual nos. are given only up to V_7 – The others become so large that at least 12 additional digit places would be required to print them out. Also in

the table was given the no. of digits in the no. of odd prime divisors — the perfect numbers known are even and it is not even known whether any odd perfect numbers can exist

A multiperfect no. like 120 gives $\sigma(120) = 240$, but multiperfect nos. include 120 also as a divisor ie $\sigma(120) = 240 + 120 = 360$ & is called a triperfect number. There are 550 multiperfect nos. known so far

Class	No. of multiperfects
Tri-perfect	6
Quadri-	36
Quinque-	62
Sexi-	222
Septi-	217
Octi-	7
	= 550

Unlike perfect nos. no known formula for multiperfect numbers. The huge size of the large perfect nos. precludes any "rand work". On the other hand Alan. L. Brown, one of the last of the "human computers" of Montclair, New Jersey, has devoted more time and energy to the study of multiperfect numbers than any one else has tabulated all of them. A number of these were discovered by him without the aid of electronic computers

Questions still requiring answers are: Are there any more multiperfect numbers in the classes already known? Are there any multiperfect nos. beyond the 8th class? Are there any odd perfect or multiperfect nos.

(b) Prime numbers — Since there are an ∞ no of primes, computers can never complete the task of finding & listing all of them — Even to determine the number of primes less than 10^{100} would tax the abilities of the fastest known computer — Prime no. theorem says no of primes less than $N \to \frac{N}{\log_e N}$ as N becomes larger & larger — other formulas become more complicated as they become more accurate.

(i) ~~(a)~~ Repunit primes - these consist entirely of ones. i.e. 11, 111, 1111, 11111, 111111, 1111111, 11111111, 111111111, 1111111111, 11111111111, 111111111111, 1111111111111, 11111111111111, 111111111111111, 1111111111111111, 11111111111111111, 111111111111111111, 1111111111111111111, 11111111111111111111, 111111111111111111111, 1111111111111111111111, 11111111111111111111111, 111111111111111111111111, 1111111111111111111111111, 11111111111111111111111111, 111111111111111111111111111, 1111111111111111111111111111, 11111111111111111111111111111, 111111111111111111111111111111, 1111111111111111111111111111111, 11111111111111111111111111111111, 111111111111111111111111111111111, 1111111111111111111111111111111111, 11111111111111111111111111111111111, 111111111111111111111111111111111111, 1111111111111111111111111111111111111, 11111111111111111111111111111111111111, 111111111111111111111111111111111111111, 1111111111111111111111111111111111111111, 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known is a 12-term one beginning with 23143 and ending with 353473 having a C.D = 30030. Construction of a 30-term sequence remains technical problems of extremely high order & this is also a theoretical problem for eg. such a 30-term sequence may not exist

(VI) Primes like $n^n + 1$ — If $n = 1, 2$ or 4 the primes 2, 5, 257 result. No others are known. It has however been shown that if there be any other primes of this form they must consist of more than 300,000 digits. Similarly if primes of the form $n^{n^n} + 1$ exist (besides $n = 1, 2$), then such primes must have at least 10^{18} digits

The formulation of prime-number problems is considerably easier than their solution. If prime numbers are found to have some degree of regularity or predictability, perhaps progress could be made. Perhaps, too, they might lose some of their fascination.

(c) Amicable numbers — only about 40 amicable pairs are known (for eg. smallest being 220 and 284). One method of constructing amicable pairs: Let

$$a = (3)(2^x) - 1$$
$$b = (3)(2^{x-1}) - 1$$
$$c = (9)(2^{2x-1}) - 1$$

If $x > 1$, and a, b, c are all primes, then $2^x ab$ and $2^x c$ constitute an amicable pair. For eg. for $x = 4$, $a = 47$, $b = 23$, $c = 1151$ we get the two amicable nos. 17926 and 18416. (easily verifiable) — Amicable triplets (N, N_1, N_2) i.e. $S(N) = N_1 + N_2$ where $S(N_1) = N + N_2$ and $S(N_2) = N + N_1$ — Example of triplet: 103340640, 123228768 and 124015008.

(d) Digital diversions — not very interesting

(e) Narcissistic numbers — "There are just four numbers, apart from unity, which are the sum of the cubes of their digits: $153 = 1^3 + 5^3 + 3^3$; $370 = 3^3 + 7^3 + 0^3$; $371 = 3^3 + 7^3 + 1^3$ and $407 = 4^3 + 0^3 + 7^3$. These are odd facts, very suitable for

$1^4 + 6^4 + 3^4 + 4^4 = 1 + 1296 + 81 + 256 = 1634$

$9^4 + 4^4 + 7^4 + 4^4 = 6561 + 256 + 2401 + 256 = 9474$

$12^2 + 33^2 = 144 + 1089 = 1233$

36 x 36
216
108
1296

81 x 81
81
648
6561

49 x 49
441
196
2401

99
99
1089

88 x 88
704
704
7744
1089
8833

100
100
10000
10100

people's telephone and what is more amusing, but there is nothing in them which appeals to the mathematician."

So wrote Hardy in his book A Mathematician's Apology. This section will, I hope, dispel the illusion that such numbers have no appeal to mathematicians, professional or amateur.

Narcissus, according to Greek mythology, fell in love with his own image seen in a pool of water, and changed into the flower now called by his name. Since this section deals with "numbers" in some sense "narcissistic numbers" and defining in those abbreviated terms, in some way, by multiplying manipulating the digits of the numbers themselves.

(i) Digital invariants - P.D.I (perfect digital invariants), are those numbers, the resultants of the n^{th} powers of their digits are equal to the number itself. Hardy's four numbers are P.D.I's of 3rd order i.e. $n = 3$.

R.D.I. If sum of n^{th} powers of digits of $N = N_1$ & such $N_1 \to N_2$ and so on leads to original N, then N is called a recurring/recurrent digital invariant - Many more P.D.I's and R.D.I's are given as examples & table of these for orders 1, 2, ... 10 given. A trend/marvel is that a 10^{th} order P.D.I should have been discovered: $4679307774 = 4^{10} + 6^{10} + 7^{10} + 9^{10} + 3^{10} + 0^{10} + 7^{10} + 7^{10} + 7^{10} + 4^{10}$

(a) Is there any connection between the order and number of invariants in that order?

(β) Are there any PDI's like this also primes?

(ii) VR-numbers: For eg $32.371 = 3^3 + 7^3 + 1^3$ is a PDI, but to call $165033 = 16^3 + 50^3 + 33^3$ a PDI is confusing, but it is a VR-number (visible representation numbers) - Exx of VR.

(α) (α) Sum of 2 squares: $1233 = 12^2 + 33^2$;
$5882353 = 588^2 + 2353^2$. & other numbers

(β) Incidentally 5882353 is a prime

(β) Differ of two squares: $2661653 = 1653^2 - 266^2$
& many other numbers

$$\Delta_{90} = \frac{1}{2}(90)(91) = 45 \times 91 = 4095$$

$$\Delta_{415} = \frac{1}{2}(415)(416) = 415 \times 208 = 90415$$

$$18^n = 5832 \quad (n?)$$

$18 \times 18 = 324 \times 18 = 5832 \Leftrightarrow n = 3$

$31^2 \times 325 = 961 \times 325 = 312325$

$73 \cdot 9 \cdot 42 = 657 \times 42 = 27594$

$3942 \times 7 = 27594$

(γ) Sum of $2\Delta^n$ nos is nos of the form $\frac{1}{2}n(n+1)$ or Δ_n

$585\,910 = \Delta_{585} + \Delta_{910}$ — other examples

(δ) Sum of 2 cubes. $22185 = 22^3 + 18^3 + 59^3$ — other examples

(ε) Amicable pairs: $3869 = 62^2 + 05^2$, $6205 = 38^2 + 59^2$

$57165 = 77^2 + 00^2$, $77006 = 59^2 + 65^2$

(ζ) Sum of factorials: $1 = 1!$, $2 = 2!$

$145 = 1! + 4! + 5!$

$40585 = 4! + 0! + 5! + 8! + 5!$ $\quad 0! = 1$

A thorough search by a computer showed that there are only such nos that exist.

(η) Sums of powers: $81 = (8+1)^2 = 9^2$

$4913 = (4+9+1+3)^3 = 17^3$.

$340122\,24 = (3+4+\cdots+4)^6 = 18^6$.

— Search for such cases — Laborious work

(θ) Miscellaneous: $43 = 4^2 + 3^3$, $63 = 6^2 + 3^3$

$135 = 1^1 + 3^2 + 5^3$ $\qquad 1306 = 1^1 + 3^2 + 0^3 + 6^4$

$175 = 1^1 + 7^2 + 5^3$ $\qquad 1676 = 1^1 + 6^2 + 7^3 + 6^4$

$518 = 5^1 + 1^2 + 8^3$ $\qquad 2427 = 2^1 + 4^2 + 2^3 + 7^4$.

$598 = 5^1 + 9^2 + 8^3$ $\qquad 31435 = 3^3 + 1^4 + 4^3 + 3^5$

The last example 3435, is the only one of its type known. and can the reader find any other number such that the sum of its digits raised to their own past power is equal to the original number? The example shown is the only 4-digit case possible

(ι) Multigrades: $1^n + 6^n + 8^n = 2^n + 4^n + 9^n$ $\quad (n = 1, 2)$

$1^n + 5^n + 8^n + 12^n = 2^n + 3^n + 10^n + 11^n$, $(n = 1, 2, 3)$

(κ) Miscellaneous. To find an integer whose cube would be represented in 5 different ways as the sum of the cubes of 3 +ve integers ie find $N^3 = a^3 + b^3 + c^3$; (a, b, c) 5 distinct sets. Table of Examples

$N = 492$	$N = 792$	$N = 870$
a, b, c	a, b, c	a, b, c
$(24, 204, 480)$	$(30, 456, 738)$	$(17, 687, 694)$

In each

$$\begin{array}{r} 125\times125 \\ \hline 625 \\ 250 \\ 125 \\ \hline 15625 \end{array}$$ } not automorphic

$$\begin{array}{r} 25\times25 \\ \hline 125 \\ 50 \\ \hline 625 \end{array}$$

$25^2 = 625$

So 25 also must also be an automorphic number. } * ?

$73\cdot9\cdot42 = 73\times378$

$$\begin{array}{r} 1134 \\ 2646 \\ \hline 27594 \end{array}$$

$$\begin{array}{r} 3942\times7 \\ \hline 27594 \end{array}$$

$2^5\cdot9^2 = 81\times32$

$$\begin{array}{r} 162 \\ 243 \\ \hline 2592 \end{array}$$

* only two automorphic pair numbers have any given number of digits and these are intimately related. Except for final digits, the sum of the corresponding digits in the pair sums to 9

$2^5 9^2 = 32\times81$

$$\begin{array}{r} 162 \\ 243 \\ \hline 2592 \end{array}$$

$$\begin{array}{r} 31\times31 \\ \hline 961\times325 \\ \hline 4805 \\ 1922 \\ 2883 \\ \hline 312325 \end{array}$$

$$\begin{array}{r} 425\times81 \\ \hline 425 \\ 3400 \\ \hline 34425 \end{array}$$

$$\begin{array}{r} 169\times343 \\ \hline 509 \\ 676 \\ 507 \\ \hline \end{array}$$

(g) Factorial products — Here factorials are the products of one in A.P.

At present only 5 solns are known

$(0!)(1!) = 1!$ $\quad (1!)(3!)(5!) = 6!$

$(1!)(2!) = 2!$ $\quad (1!)(3!)(5!)(7!) = 10!$

$(6!)(7!) = 10!$

If not necessarily in A.P, there are $(4!)(23!) = 24!$

$(2!)(4!)(47!) = 48!$

$(2!)(3!)(4!)(287!) = 288!$

(h) Printers errors — Given by Dudeney in his Amusements in Maths, stating a printers error to be correct

$2^5 9^2 = 2592$

Vandeport notes a number other cases:

$2^5 \cdot \frac{25}{31} = 25\frac{25}{31}$,

$11^2 \cdot 9\frac{1}{3} = 1129\frac{1}{3}$

$21^2 \cdot 4\frac{9}{11} = 2124\frac{9}{11}$

Many solns can be used to generate an ∞-number of errors

2 other cases: $73 \cdot 9 \cdot 42 = 7 \cdot 3942$

$73 \cdot 9 \cdot 420 = 7 \cdot 39420$.

(i) Automorphic numbers — is one whose square ends with the given number: Ex: $5^2 = 25$, $76^2 = 5776$, $625^2 = 390625$

(ie 5, 76, 625 are automorphic *)

Hunter found a pair of 17 digit automorphic numbers. (i.e. a 167-digit pair has been found †

3953007 — — — — — — 212890625

6046992 — — — — — — 787109376

The decimal automorphic nos. check when cut by squaring either one of them to see if the product terminates with the original number

There are only 167 different automorphic nos. known at present

(j) Two rather large numbers

(1) What is the largest number which can be written by using only 3 digits? It turns out to be $9^{99} = 9^{387420489}$.

1077

569
59
449
1077

The number if calculated and written out at 5 digits to an inch, would take nearly 11.67 miles of paper strip. However this fantastic number can never be fully evaluated. However, at the Rand Corporation Fred Gruenberger calculated the last 1000 and first 1200 digits of this number! Here are the first and last 10 digits

$$4281247731 \ldots \ldots 2627177289.$$

(ii) one more large number:

$$9^{9^9}$$

A calculation shows that this number if written out in full at 5 digits to an inch, would take about $1.16 \times 10^{369693094}$ miles of paperstrip. Even if the ink used in forming this number were in a layer but 1 atom thick, there would not be enough matter in millions of our universes to accomplish this task. Nevertheless, with a knowledge of the elementary properties of numbers and a simple desk calculator, the last ten digits of this stupendously huge number have been calculated

$$\ldots\ldots 1045865289.$$

9|3|31 New Books section

(1) ✓ Documents on Modern Physics – Dynamical Theory of Groups and Fields by Bryce S. DeWitt – Methods of metrical of a theoretical nature.

Came back to "Science" after a long time. But switches on to Proc. Roy. Soc.

(2) P.R.S. A. Vol 318, pp. 243-399, No. 1534 – 8|9|70

✓✓ A discussion on duality – Regge poles and resonances in elementary particle processes

organised by R. H. Dalitz

– Specially the Glossary on pp. 248 – 255 [illegible] helpful in understanding the subject

On the last page (p. 39 f, in the discussion H. Harari (Weizmann Institute of Science, Rehovot, Israel) remarks " I would like to mention one point which, I think, should be made after discussing duality for a whole day. A person who has not followed the development of particle physics during the last ten years would probably think that we are all completely out of our minds, since we have not mentioned the words 'one pion exchange' (O.P.E.) even once today. The one pion exchange model, including its Reggeized version, is of course still with us, and we do believe that pion exchange dominates in most reactions to which it can contribute. Whether you Reggeize, absorb, conspire, evade or whatever you like to do with exchanged pions, in one way or the other the O.P.E. term exists. A very interesting question which has not been answered yet is: what is the duality character of the pion? what is the role played by O.P.E in the duality scheme? The difficulty is, of course, that π exchange amplitudes are always predominantly real while duality makes sense only for the imaginary parts of the amplitudes. The pion exchange term is not really understood from the s-channel point of view, except for the ancient (but correct) observation that the π exchange term involves contributions of many s channel partial waves. It would be interesting to understand whether the pion term is just some real constant (like a subtraction term) or it has some clearer connection with s channel effects

(In the Glossary) under relating to Regge Trajectory — Plot of the name given to the line $J = \mathrm{Re}\alpha_J(t)$ [or $J = \beta_J(M)$] or a plot of J against $\frac{1}{t}$ or $\frac{1}{M^2}$ on a Chew-Frautschi plot. Each trajectory corresponds to definite values for the quantum numbers for isospin, strangeness, baryon number, parity (and

Charge-conjugation family or G-family in the case of mesons), and for the unitary symmetry pattern $\{a\}$. For a meson trajectory the pseudoscalar meson states correspond to the spin values $J = 0, 2, 4, \ldots$ or $J = 1, 3, 5, \ldots$, the intermediate states being absent owing to the signature factor in equation (2) [p. 240 of Dalitz's Introduction]. For a baryon trajectory the physical states on a trajectory have $J = \frac{1}{2}, \frac{5}{2}, \frac{9}{2}, \ldots$ or $J = 3/2, 7/2, 11/2, \ldots$ depending on the signature of the trajectory

(3) P.R.S. 318, No. 1535 pp. 407-544, 22/9/70

pp. 441-457 - Studies of non-linear phenomena I - Theory of the hyper Raman effect by D. A. Long & L. Stanton.

13/7/71 New Books Section.

(1) The Atom - The Marshall Cavendish Learning System: Physical Science (1969) - The book may not be sold in the U.S.A or Canada (I wonder why?) - No mention of Raman effect in the Index.

- Mention of protons attracting electrons & such electrostatic forces being explained by exchange of photons - photoelectric effect viz. photons hitting a metal & knocking out electrons giving an electric current - Compton effect viz. when beams of X-rays are scattered by a slab of carbon their scattering is accompanied by a movement of electrons.

In the future it may well be that the intensely powerful beams of lasers, which show up one million times brighter than the brilliance of our Sun, could well be used to transmit interstellar messages. But light-borne conversations across space will always be lengthy affairs. Even at the speed of light a reply from Alpha Centauri, our nearest star, could not be received until a eight and half years after a question reached it from the Earth.

(2) Sources of Quantum Mechanics - edited by with a historical intro by

* electromagnetic processes can be understood as the interaction of two currents.

4 Wiss. z. Zwick

by van der Waerden (North-Holland, 1967) – Publication of the earliest papers on Quant. Mech. are ideas due to originally to Max Born – lots of references to Max Born in the book – Good historical material – may be useful if I ever write something about Max Born later. –

p. 19 – The birth of quantum mechanics – Born's recollections of the scientific atmosphere at Göttingen during 1923-25 – Also pp. 19 - 59 of the Introduction by van der Waarden is quite interesting

p. 20 – "The word quantum mechanics appears for the first time in physical literature in a paper of mine (über Quantenmechanik. Z. Phys. 26, p. 379, received June 13, 1924) in which an essential step towards the establishment of this new doctrine was made.

p. 36 – Born's conjecture on pq - qp. – Giving Born's own account. – Pauli's reaction (p. 37)

p. 42 – The "three men's paper" by Born, Heisenberg & Jordan

~~Zeit~~. Z. Phys. 35, p. 557, recd. Nov. 16, 1925.

Science:

3/4/70 – p. 104 – Parity-violating nuclear forces by G. C. Wick. – A historical up to date account (emphl) finishing Feynmann & Gellmann's ~~over~~ theory "Universal Theory of weak interactions" [Phys. Rev. 109, 193 (1958)] modelled after the electromagnetic theory* and assuming that weak nuclear phenomena could also proceed by way of a current-current interaction. Nuclear beta decay can be described as the interaction of a nucleon current with a lepton (e & neutrino) current. The interaction of the lepton current with a strange particle current allows for lepton decays of strange particles, and so on. This "Universal Theory also provides for the interaction of a current with itself, and the case of the interaction of a nucleon current with a nucleon current gives rise to parity violations in nuclear forces.

3/7/70 - p. 20 - Laser Raman Spectroscopy by A. Mooradian - Quite an interesting paper

p. 26 - The bridge of values by Barnaby C. Keeney - Adapted from an address delivered presented on 28/4/70 before the National Academy of Sciences, Washington D.C. - A humanist's view - Talks of the legend that the "U.S. never lost a war, and never won a peace".

p. 34 - A small article by Samuel Z. Goldhaber on "Student - Student polarisation plagues U.S. Campuses" throws light on the suicide yesterday of a student at the Institute.

p. 40 - Experimental tests of relativity by R.W. Holcomb - Talks of "Black holes", 'gravitational collapse', big bang theory & so on

24/7/70.

p. 350 - Transit of the planet Mercury. - 12 such transits during the 20th Century. Next one will be on 10/11/73. These transits, oddly enough, can only take place within 5 days before or after 8th May or within 5 days of 10th November, when the Sun passes the nodes of Mercury's orbital path. Usually this event occurs 13 'about 15 times each century. The importance of these transits are that they can be precisely timed and used as a test of the time system employed by mankind on earth. The transits of Venus are even more important for this same purpose, but these are less frequent. None will occur during the 20th Century. The last transit of Venus was in 1882 and the next will not occur until 8/6/2004.

p. 360 - Review of books by Max Born (3 books) with the heading Max Born on his Vocation by Martin J. Klein - most interesting - Perhaps I have already seen these - The last of the three books consists of correspondence between Born, his wife & Einstein, published in 1969 (Did Born write to me about the book?)

Bertrand Russell's brief preface to the last book is very moving — The other two books also are in 1969 & 1968.

31|7|70.

7|8|70. p. 554 — A scientific safari to Africa by G.T. Seaborg
p. 565 — Brain drain; fewer scientists enter U.S., More seek to leave by T. P. Southwick.

14|8|70. ✓ p. 633 — Editorial — Science: Attack and Defense.
— The only effective defense of science is through strengthening science itself — — our continued and lively awareness of useful & humane applications, our careful and responsible use of funds — these will maintain respect for science & scientists. Then we can say with Brutus

" There is no terror, Cassius, in your threats
For I am arm'd so strong in honesty
That they pass me by as the idle wind"

— As cited by Kenneth V. Thimann, Univ. of Calif, Santa Cruz.

p. 654 — Behavioral sciences: The view at the Centre of Advanced Study, Stanford — Arsonists setting brush fires in the hill slopes of this centre which stands on an isolated hilltop on Stanford Univ. land. The heaviest damage done to research materials was sustained by a sociologist from New Delhi (M.N. Srinivas), p. 657 for details

✓ 4|71 21|8|70. p. 721 — Editorial on "Science and immediate social goals" by Philip. H. Abelson —

Harvey Brooks pointing out that there is ample evidence to show that it takes about 10 years to arrive at a technological goal having significant impact on Society — The last sentence is very very appropriate in connection with the planning efforts in India today — " Thus, incessant efforts to make science more productive in terms of immediate social goals may make it

* further extracts from the Report of the National Goals Research Staff

(U.S. Govt. Printing Office, Washington D.C., 4/7/70)

"To the extent that society insists that basic scientists do work that is more relevant to present social needs ..... scientists will be less able to work where nature appears willing to answer their questions. They may be required to work on relevant questions that perhaps cannot be answered at all at present, or can be answered only with uneconomic use of resources".

far less productive in the long run.*

p. 733 – A theory of the origin of the State.

p. 739 – Communication or Chaos – Effective transfer of scientific and technical information continues to be a pressing national problem.

p. 743 – Science & Politics: Free speech Controversy at Lawrence Laboratory.

28/8/70. p. 819 – Editorial by Robert S. Morrison, Div. of biological sciences, Cornell Univ., Ithaca, N.Y. on

"Agnew, Alcohol, Automobiles and Assessment" – Criticism of drunken automobile drivers causing 50,000 deaths a year in the U.S

4/9/70. p. 927 – "Hysterical Advice" – characterising the request of 44 Nobel laureates to President Nixon to take decisive steps towards the termination of the Viet-Nam War.

Rational turn to an oracle

p. 933 – The fundamental nuclear interaction by A.E.S. Green. – Important developments of the past decade on mesons and the theory of nuclear forces are described – Very. Comprehensive and interesting account introducing the ω, ρ and η-mesons. [Ref. to L. W. Alvarez. Science 165, 1071, (1969).] – Also mentions the scalar meson ε, and the unitarised Veneziano model, one of the latest developments in elementary particle theory – mesons as bound states of quarks & antiquarks – OBEP group (one-boson exchange potential) & GOBEP (generalises one-boson exchange potential. –

p. 941 – Article on Birth-Control after 1984

p. 964 – Review of a book "Birth-Control in America".

p. 974 – A radar snapshot of Venus – A radar brightness map of Venus has been obtained. It reveals interesting surface features and much structure over a large area – 3 fine radar maps given.

2/4/71 In the Journal Universitas (a German Review of Arts and Sciences – Quarterly English Language Edition) – Vol. 12, No. 4, 1970.

√√√ || 11-287 – In Commemoration of Max Born by Prof. Werner Heisenberg, Munich. – Read it through & found it very interesting.

5/4/71 from Sierpinski

Fermat Numbers – For F_5 & F_6 we know the prime factors.

For F_{1945} we know it is composite but factors not known one factor viz $5 \cdot 2^{1947}+1$ (587 digits) is known. only proved theoretically No. of digits in $F_{1945} > 10^{582}$ & hence impossible to write all the digits. Also $m = 5 \cdot 12^{47}+1$ is prime & is smallest divisor of F_{1945}.

For F_7, F_8, F_{13} & F_{14} we do not even know the decomposition as products of two integers > 1 although we know that such a decomposition exists.

For F_9, F_{10}, F_{11} & F_{12} we know a prime divisor

F_{17} has 30 thousand digits & existing computers cannot check its primality or otherwise

For F_{16} we know a smallest prime divisor $2^{18} \cdot 3150+1$

—

(1) Simple examples of no A.P. having four prime terms under 300 digits

251, 257, 263, 269.

p. 1001 – Single atom visibility – enhancement of resolving power, with a scanning electron microscope, single uranium and thorium atoms – A similar feat of seeing gold atoms
– The Venus radius controversy

11/9/70 p. 1031 – United States Goals in Vietnam – No. of Letters

p. 1053 – Science under Nixon: Influence has declined in National Affairs

p. 1058 – Univ. of California: Political & financial woes

p. 1061 – Campus unrest: which tack for Presidents Commission?

4/4/71

New Books Section.

(1) Mathematical Essays for Cosmics – A collection of essays by 21 eminent mathematicians Edited by Committee of Support of research in the mathematical sciences (Conf. COSRIMS)
[meant ~~to~~ Intended not only for the non-mathematical Scientist but also for the scientifically oriented layman]

p. 115 – At the end of Dyson's Scientific American Article reproduced here is given a short biographical sketch of Dyson which appears almost to resemble mine in a pure mathematician turning towards theoretical physics.

p. 116 – Essay by A. S. Wightman on "Analytic functions and elementary particles" is quite interesting and talks of piecewise smooth and piecewise analytic functions (referred to by E.C.G at the P-M-Mathews Conference at Madras)

[Wightman at Princeton is a member of both the math. & phys. Departments. His emphasis has since interest as follows:– 1) a mathematical elem. particle physics theory 2) realizing that its mathematical structure is of great mathematical complexity. Since being along with like minded people, I have used everything at my disposal, the intuitive sense in trying to

Waring's Conjecture – Every integer n is the sum of the k^{th} powers of $g(k)$ integers

F. V. Andrica [Proc. Sem. Brasil II (1980) pp. 437–451] has shown that every integer $> 10^{10^{89}}$ is the sum of nineteen biquadrates

[I don't think I should mention either this or Pillai's work here]

Some simple problems from Sierpinski (out of his 100)

(2) Give two examples of primes of the form x^4+1

$(17 = 2^4+1,\ 257 = 4^4+1)$ – not known if there be ∞^{ly} many

(3) Three successive primes in A.P (i.e. no primes between them)

$(3, 5, 7;\ 11, 17, 23;\ 47, 53, 59)$

(4) Find an integral solution of the equation (other than $x = y = 1$)

$$2x^4 - y^2 = 1$$

[$x = y = 1$, and $x = 13,\ y = 239$]

(5) Find a value of n for which $2^n - 7$ is prime

[$n = 39$]

(6) Give an example of a triangular number which is also tetrahedral

[If $P_n = \frac{1}{6}n(n+1)(n+2)$ be the n^{th} tetrahedral no.

and $t_n = \frac{1}{2}n(n+1)$ " Δ^{r} no

$P_1 = t_1,\ P_3 = t_4,\ P_8 = t_{15},\ P_{20} = t_{55},\ P_{34} = t_{119}$]

(7) Sierpinski's final remarks: The no. of unsolved problems continually increases with time. This is because the new problems arise more rapidly than those already established are solved, and plenty of them have remained unsolved for centuries (*). But the progress of our knowledge of numbers is attained not only by what we already know about them, but also by realising what we yet do not know about them

(*) When I said this in a lecture, a prominent mathematician said "certain problems will therefore be never solved". I answered that this was of course possible, but it is unusually unlik

understand this structure.

(2) Symmetries and Reflections - Scientific Essays by E. P. Wigner (Indiana Univ. Press, Bloomington & London, 1967)

p. 252 - Reflections about Fermi

p. 257 - " John von Neumann.

(3) An autobiography of Timoshenko:

Science (continued)

~~6~~/9/70 ~~An article~~ On the usefulness of lunar science - Editorial by P. Cloud, p. 1159.

/9/70 p. 1292 - 15 top economists oppose SST (Supersonic transport project)

/10/70. ———

/10/70. p. 125 - Editorial by S. F. Singer on "Will the world come to a horrible end? " . . As scientists we have the responsibility to speak up, but we also must know when to stop talking"

pp. 139-40 - Interstellar molecules; Chemicals in the sky by G. L. Wick

" Despite all these unanswered questions, radio-astronomical observations of interstellar molecules have provided astrophysicists with solid data for their theories of stellar cosmogony"

6/10/70. p. 265 - Editorial - Review of a the book by Vannevar Bush " Pieces of the Action' (Morrow, N.Y. 1970)

p. 311 - "Basic Mechanisms of the Epilepsies" based on a Symposium (Eds. Little, Brown, Boston, 1969) - reviewed by H. G. Vaughan [Dept of Neurology, Albert Einstein College of Medicine, N.Y. City]

3/10/70 · pp. 422-24 - Account of the three winners of the Nobel Prize for Chemistry for 1970 - Von Euler, Julius Axelrod, B. Katz

last infinitely long, the situation might be paradoxical — every problem would eventually be solved. In fact, suppose that ten new problems are posed every year and only one of the already posed ones is solved. Already the number of unsolved problems will increase indefinitely, namely after n years there will remain $9n$ unsolved problems. But if the new problems posed are marked with successive numbers and each year the problem marked with the lowest number (of those already posed) is solved, then the n^{th} problem will be problem after n years, and thus in time every one of these problems will be solved.

p. 425 – Ion implantation by G. d. Wick.

3/10/70 p. 514 – Antiwar group raises $ 250,000.

p. 520 – Deep sea drilling – A giant step in Geological Research.

p. 535 – Mercury – the dark-side temperature – Has suggested that Mercury and the Moon have very similar top layer surfaces.

/11/70. p. 578 – Recalcitrant honey bees (apiology)

p. 587 – Editorial on Doctor of Arts Degree

p. 604 – Nobel Prizes, 1970 awards honours three in Physics & Chemistry

Born 1908) (i) Hannes Alfvén* – Swedish iconoclast recognised after many years of rejection & obscurity – Note by A. g. Dessler

(ii) French physicist – Louis Néel – fundamental work on ferromagnetism & anti-ferromagnetism leading to important applns in solid state physics

Chemistry – Louis Leloir the Argentinian Biochemist

3/11/70.

0/11/70 p. 840 – Pieces of the Action by V. Bush reviewed by D. Stewart

80/11/70 p. 960 – Ultraviolet Astronomy

p. 962 – Spectroscopic Astrophysics – contributions of Otto Struve. – (G. H. Herbig) – A memorial volume to a great astronomer

12/70 p. 1183 – Review of the book "Geometrical Thinking – Space through the ages by C. Lanczos (Academic Press, N.Y. 1970)

first para of the Review: When Einstein, in 1928, was looking for a capable assistant, he happened to meet a 35-year old Hungarian mathematician who was working at the Univ. of Frankfurt – a Mr. Klein. Impressed by the mathematician's knowledge and personality, Einstein invites him at once to work on certain problems on general relativity. At the first

session Einstein asked him to solve a new type of field equations so that certain conditions would be satisfied. Bond, on the assignment, Einstein's new collaborator got deeply absorbed in the problem and after three or four days, to his greatest satisfaction, found the required solution. At the appointed hour, happy at his success, he showed Einstein that the solution indeed met all the conditions required. "Quite remarkable" replied Einstein, adding after a short pause silence, "But don't you see, that I gave you the wrong equation".

Lanczos' optimism that, although Einstein's efforts, in formulating a unified theory accounting for all physical effects on the basis of geometrical conceptions alone have not been successful "this program will be tried again and again, until one day full victory will come?" This optimism appears to stem for his intimate collaboration with Einstein.

p. 1196 – How did Venus lose its angular momentum? by S. F. Singer

Abstract: Venus now has a retrograde and negligible spin, but it is very likely started with a typical planetary spin: prograde and with a 10- to 20-hour period. The usually assumed mechanism of solar tidal friction is quite insufficient to remove this angular momentum. Instead, we postulate capture of a moonlike object from an initially retrograde orbit: it would despin Venus and suddenly transform the planet's rotational Kinetic energy into internal heat, which would lead to volcanism, and the liberation of large amounts of volatiles. The moon could disappear by crashing into the surface of Venus.

Simple mathematical treatment.

12 | 70 p. 1263 – The Univ. & the problem of education by E. H. Levi, Office of the President, Univ. of Chicago.

Universities are among the most important institutions in our Society, but there are other important institutions. You

will recall de Tocqueville's description: "Americans of all ages, all conditions, and all dispositions constantly form associations. They have not only commercial and manufacturing companies, in which all take part, but associations of a thousand other kinds, religious, moral, serious, futile, general or restricted, enormous or diminutive" - - - A University cannot take on all these functions and it would be deceitful or foolish or both to assume so - A University should lay emphasis on research which is paramount. Research & teaching to be closely related. Research itself - the restructuring of subject matter, the revelation of insights new and old - can be the highest form of teaching. The University should give the individual scholar the freedom and intellectual environment required for research, and ~~shall also undertake~~ also to undertake to transmit the qualities and understanding of research into all of its teaching. The greatest service of a Univ. is in its commitment to reason, in its search for basic knowledge, in its mission to preserve and to give continuity to the values of mankinds many cultures. In a time when intellectual values are denigrated, this service was never more required

p. 1283 Glenn. T. Seaborg, Chairman AEC wins the election to Presidentship of the AAAS (Amer. Assn. Adv. Sci)

p. 1297 Summary of arguments for a ~~hot~~ hot moon by R. B. Baldwin

The deep moon must be hot, although not necessarily molten. The surface is cool, and we may infer a transition zone in the outer few hundred kilometers

~~Some~~ [Some scientists even today consider the moon to be cold or cool inside]

12/70. p. 1358 - More than two-thirds of U.S. research and development is accomplished in the laboratories of private companies --- the American private sector, ~~has to its credit~~ it should be underscored, has a proven record for assembling unite -

disciplinary teams of outstanding individuals to solve the most pressing technological challenges in all history. Let us enlist this expertise for public sector too.

p. 1359 – How to approach dwindling funds ? by W. P. Jacobs. – Very interesting remarks re. submission of 20 copies of each research project (like the CSIR wanting 40 copies !)

p. 1361 Editorial – Science on a tight budget by Brian H. Flowers, Chairman, Science Research Council, London.

Remark of J. M. Ziman quoted " In recent years, it has become just a bit too easy to get a Ph.D of sorts, and an academic job of sorts, and a grant for research of sorts – to produce yet more Ph.D's of the same sort".

We insist that we shall always support outstanding scientists, whatever they do and wherever they work, and shall always provide funds for trying out good new ideas.

p. 1363 Galaxies and the Universe by J. H. Oort

p. 1431 – Criticism of some work of Alfven (Mar an. IV. 2) and G. Arrhenius on " Asteroid Lunacy" by Joseph. H. Guth and their reply.

15/1/71. p. 151 – Addiction as a necessity and an opportunity by E. H. C.

A magnificent meaningless lecture about Super-man and "Multiple-man" and " singular man" – Addiction means " making at least one good explor. a day ie Concentrating on one topic & finishing it" – Multiple-man interested in all kinds of things.

This lecture stirs a multiple-man

Science Progress (Review Journal of current scientific advance

p. 507 – Broken symmetry* by G. E. Stedman (Research Instt. Dept. of Physics, Queen Mary Coll. London, E. 1)

– Concept of B. S* may provide a fruitful link between the theories of the astronomical & sub-atomic worlds

$2^{11} - 1 = 2047 = 23 \times 89$ //